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CAUSALITY, NONLINEAR SUSY AND INFLATION

Quentin Bonnefoy (DESY-TH), Gabriele Casagrande (CPHT-Ecole Polytechnique), E.D., [arXiv:2206.13451 [hep-th]]

and also

E.D., M.A.G.Garcia, Y.Mambrini, K.A.Olive, M.Peloso and S.Verner, Phys. Rev. **D103** (2021), 123519 [arXiv:2104.03749 [hep-th]]

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Outline

1) Spin 3/2, potential problems
2) Gravitino sound speed in supergravity
3) Causality and positivity bounds
4) Alternative minimal models of inflation
5) Perspectives

1) Spin 3/2, potential problems

- In supergravity, the gravitino Ψ_μ becomes massive by absorbing the goldstino G

$$\Psi_\mu \begin{pmatrix} 3/2 \\ - \\ - \\ -3/2 \end{pmatrix} + G \begin{pmatrix} - \\ 1/2 \\ -1/2 \\ - \end{pmatrix} = \Psi_\mu \begin{pmatrix} 3/2 \\ 1/2 \\ -1/2 \\ -3/2 \end{pmatrix}$$

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The consistency of low-energy actions for the spin 3/2 Rarita-Schwinger field has a long history :

- 1941: Rarita-Schwinger action
- 1969: Velo-Zwanziger pointed out potential acausal propagation for a charged gravitino in an e.m. background
- 1977: Deser-Zumino proved that gravitino propagation in minimal supergravity is causal
- 2001: Deser-Waldron proved that gravitino propagation in gauged supergravities is causal

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- 2021 – Gravitino swampland conjecture, gravitino mass conjecture (Cribiori,Lust,Scalisi; Castellano,Font, Herraez,Ibanez, 2021)

History strongly suggest that usual supergravities have no problems with gravitino propagation.

SUSY/SUGRA (linearly realized): nb. bosons = nb. fermions
Nonlinear SUSY/SUGRA: nb. bosons $\neq$ nb. fermions

Inflation models in standard SUGRA's have at least one complex scalar field (often several).
Recently, simple nonlinear SUGRA models were constructed.
More minimal inflationary models, fewer fields.
(Antoniadis,E.D.,Ferrara&Sagnotti; Kallosh,Linde & coll, 2014)

Possible to construct minimal models with only:
graviton, massive gravitino and inflaton (real scalar)

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Simplest nonlinear SUSY's: constrained superfields (talk F. Quevedo).

Why non-linear SUSY ? Anti D3 brane in KKLT, strings with broken/nonlinear SUSY, split SUSY…

(Rocek,78) introduced a constrained, nilpotent superfield

$$S^2 = 0$$

whose solution is

no fundamental scalar

Superspace fermionic coordinate

$$S = \frac{GG}{2F_S} + \sqrt{2}\theta G + \theta^2 F_S$$

auxiliary field

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2) Gravitino sound speed in supergravity (SUGRA)

The talk deals with the « speed of sound » c_s of gravitino in SUGRA, in inflation and more general time-dependent sols

Normally $0 < c_s \leq 1$

Recently, two potential problematic behaviours were discussed:

- $c_s = 0$ at particular points on the inflationary trajectory

$\Longrightarrow$ Large (catastrophic) production of gravitinos

(Hasegawa, Terada et al, 2017; Kolb, Long, McDonough, 2021).

- $c_s > 1$ acausal behaviour at particular points on the inflationary trajectory in specific SUGRA models

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The sound speed c_s is defined from the dispersion relation

$$\omega^2 = c_s^2 \mathbf{k}^2 + a^2 m^2$$

The transverse spin 3/2 component in a FRW background has a standard dispersion relation with $c_s = 1$

For the longitudinal component:

$c_s < 1 \Longrightarrow$ Slow gravitino (Benakli,Darmé, Oz, 2014)

$c_s > 1$ possible for particular nonlinear SUGRA models with orthogonal constraint

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A general expression for longitudinal gravitino sound speed is

pressure

time-derivative

$$c_s^2 = \frac{\left(p - 3m_{3/2}^2\right)^2}{\left(\rho + 3m_{3/2}^2\right)^2} + \frac{4\dot{m}_{3/2}^2}{\left(\rho + 3m_{3/2}^2\right)^2}$$

energy density

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The explicit formula in SUGRA is

$$c_s^2 = 1 - \frac{4}{(|\dot\varphi|^2 + |F|^2)^2} \left\{ |\dot\varphi|^2 |F|^2 - |\dot\varphi \cdot F^*|^2 \right\}$$

where $F^i = \mathrm{e}^{K/2} K^{ij^*} D_{j^*} W^*$ in standard SUGRA ,

and we used the compact notation $|\dot\varphi|^2 = \dot\varphi^i K_{ij^*} \dot\varphi^{j*}$,etc

Obs: Cauchy-Schwarz inequality $\Longrightarrow$ causality $c_s \leq 1$
respected in standard SUGRA's

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For the (large) majority of SUGRA models we investigated , we found no problems : $0 < c_s^i \leq 1$

Few models with problems: ex. « orthogonal constraint » for the inflaton multiplet Φ (Ferrara,Kallosh,Thaler)

$$S(\Phi - \overline{\Phi}) = 0 \quad \Longrightarrow$$

Only $Re\ \phi$ =inflaton is a dynamical degree of freedom. $Im\ \phi$, the inflatino ψ_ϕ and the auxiliary field F_ϕ are determined by the constraint.

In particular F_ϕ is a bilinear in fermions and does not appear in the scalar potential : $F^\Phi \neq e^{K/2} K^{\Phi \bar{i}} D_{i^*} W^*$

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Consequences:

- No inflatino ⇒ gravitino sound speed problem $c_s = 0$ can arise (model-dependent)

- Cauchy-Schwarz argument for $c_s \leq 1$ not valid. Examples (orthogonal constraint) with $c_s > 1$!

However, the UV origin of the orthogonal constraint is not clear (Dall'Agata,E.D.,Farakos, 2006; Bonnefoy,Casagrande, E.D)

⇒ Potential pathological behaviour reminiscent of the swampland program ! (Vafa,Ooguri)

3) Causality and positivity bounds

(Q.Bonnefoy,G. Casagrande & E.D., [arXiv:2206.13451 [hep-th]])

- The potential acausal behaviour concerns the longitudinal component of the gravitino.
- Gravitino equivalence theorem: at high-energy, gravitino longitudinal component described by the goldstino, enhanced couplings to matter.

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General lagrangian orthogonal constraint

$$K = h(\mathcal{A})\,\mathcal{B}^2 + S\bar{S}$$

$$W = f(\Phi)S + g(\Phi)$$

where $\qquad \Phi = \mathcal{A} + i\mathcal{B}$

Is the acausality found in SUGRA captured by the low-energy lagrangian of the goldstino coupled to matter, in the decoupling limit $M_P \to \infty$?

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Yes ! Ex: The goldstino lagrangian contains a higher-derivative operator of the form

$$\frac{1}{f(\varphi)^2}\left(h(\varphi)-\frac{2g'(\varphi)^2}{f(\varphi)^2}\right)(\bar{G}i\gamma^m\partial^n G)\ \partial_m\varphi\ \partial_n\varphi$$

Operator subject to positivity constraints from dispersion relation arguments, enforce

$$\frac{h(\varphi)}{2}f(\varphi)^2 \geq g'(\varphi)^2 \quad \Longleftrightarrow \quad c_s \leq 1$$

- The issue arises due to the « elimination » of the auxiliary field by the orthogonal constraint, no simple physical interpretation.

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- Obs: SUGRA/inflation subluminality condition valid throughout the inflationary trajectory, positivity constraints valid only in the ground state

SUGRA condition is stronger.

- Causality condition of goldstino propagation in time-dependent solutions of the goldstino action seems equivalent to the SUGRA constraint.

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4) Alternative minimal models of inflation

Orthogonal constraint is « reducible » $\Longleftrightarrow$ three « irreducible » constraints (dall'Agata,E.D, Farakos,2016)

$$S\bar{S}\left(\Phi - \bar{\Phi}\right) = 0,$$ eliminates a scalar

$$S\bar{S}D_\alpha\Phi = 0,$$ eliminates the fermion

$$S\bar{S}D^2\Phi = 0,$$ eliminates the auxiliary field

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Simplest alternative with no potential acausality problems: use only

$$S\bar{S}\left(\Phi-\bar{\Phi}\right)=0$$

$$S\bar{S}D_{\alpha}\Phi=0$$

(Same) minimal spectrum for inflation :
Graviton, massive gravitino, inflaton

Equivalent alternative (Bonnefoy,Casagrande,E.D):
orthogonal constraint, but higher-derivative UV action

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Comparison

Orthogonal constraint vs Alternative constraint

$$V = e^K \{ |D_S W|^2 - 3|W|^2 \} \qquad V = e^K \{ |D_S W|^2 + |D_\Phi W|^2 - 3|W|^2 \}$$

Ex: SUGRA inflation model, alternative constraints

$$K = -\frac{1}{2}(\Phi - \bar{\Phi})^2 + \bar{S}S \quad , \quad W = f(\Phi)S + g(\Phi)$$

$$f = \sqrt{3}g \qquad g = M^2\left(\Phi + \frac{1}{a}e^{-a\Phi}\right) + g_0$$

$$V(\varphi) = M^4[1 - e^{-a\varphi}]^2$$ is the Starobinsky model

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Conclusions

- Important to check and impose sound speed
$0 < c_s \leq 1$ $\Longrightarrow$ gravitino swampland conjecture

- Most SUGRA models satisfy it, except peculiar models with orthogonal constraint (or similar).
- Subluminality constraints captured by goldstino SUSY lagrangians in $M_P \to \infty$ limit and positivity constraints, but SUGRA condition is stronger.
- Alternative minimal inflation models, no causality issues
- General interest: consistency constraints on nonlinear SUSY/SUGRA, strings with broken SUSY

THANK YOU !